ECE 172A: Introduction to Image Processing
Morphological Processing

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Outline

- Morphology: Introduction
- Basic Definitions
- Erosion and Dilation
- Opening and Closing
- Graylevel Morphology
- Morphological Filtering

Morphology: Introduction

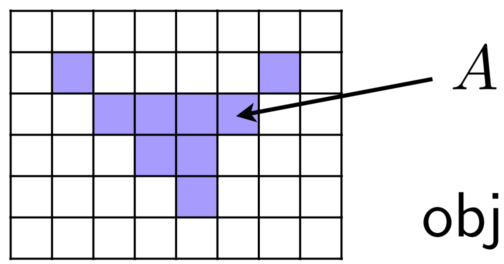
What does morphology even mean? *The study of shape and structure*

- Language: Set Theory
  - Binary images (bitmap)

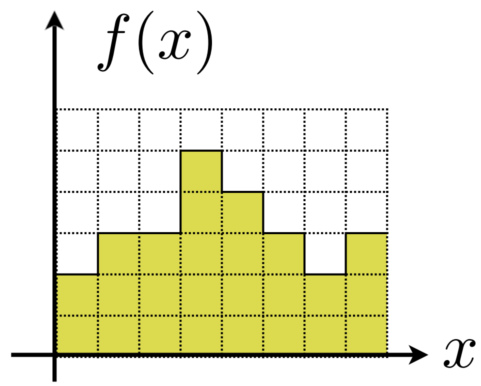
    $\Longrightarrow$ Sets of points in 2D space ($\mathbb{Z}^2$)
  - Quantized graylevel images

    $\Longrightarrow$ Sets of points in 3D space ($\mathbb{Z}^3$)

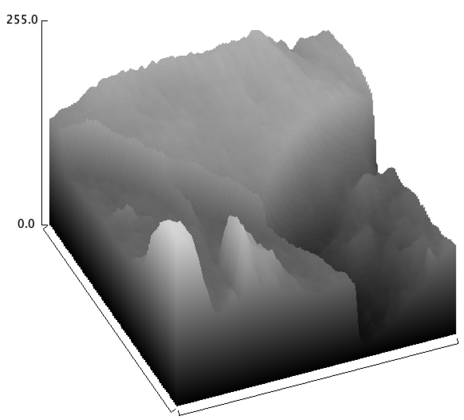
    $$(x, y, Q(f(x, y))) \in \mathbb{Z}^3$$



object vs. background



1D signal



2D image

- Types of transformations
  - Set-theoretic: Union, intersection, etc.
  - With a stucturing element: dilation, erosion

Morphology: Application Areas

Classification of objects or image features based on shape

- Examples
  - Extraction of objects with a specific shape
  - Extraction with a size smaller or greater than a limit
  - Contour detection
- Typical stages in an image-processing pipeline where it is useful
  - Preprocessing: noise reduction, simplification
  - Feature detection
  - Segmentation: Contour extraction
  - Postprocessing: shape cleaning and simplification
- Main application areas
  - Material sciences, mineralogy, granulometry
  - Medicine and biology: Cell counting, cytology, gel electrophoresis, microarrays
  - Robotics and machine vision

Basic Definitions

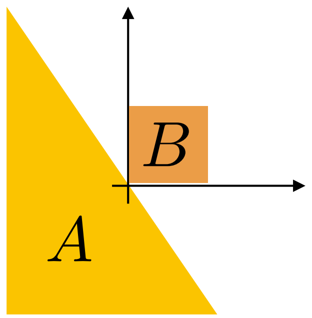
- "Universe" set

  $\mathbb{E}$ is the set of every possible element (e.g., $\mathbb{E} = \mathbb{Z}^2$ or $\mathbb{E} = \mathbb{Z}^3$ or $\mathbb{E} = \mathbb{R}^2$)

- Sets and subsets

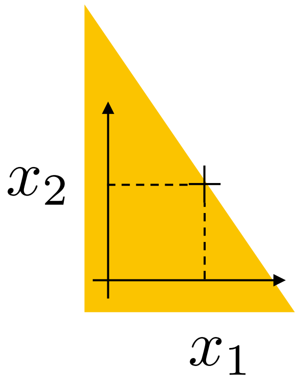
  $A, B \subset \mathbb{E}$

  Elements: $a = (a_1, a_2) \in A$, $b = (b_1, b_2) \in B$



- Translation by $x = (x_1, x_2)$

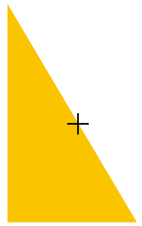
  $(A)_x = \{c \,:\, c = a + x, a \in A\}$

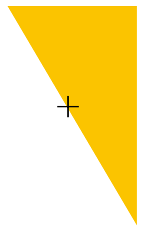


Basic Definitions (cont'd)

- Reflection or symmetry

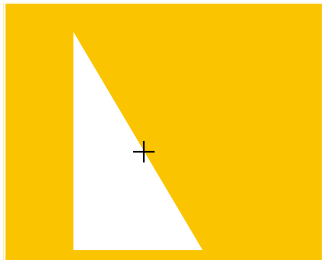
$$A^{\mathrm{s}} = \{x \in \mathbb{E} \,:\, x = -a, a \in A\}$$





- Complement

$$A^{\mathrm{c}} = \{x \in \mathbb{E} \,:\, x \in \mathbb{E} \setminus A\}$$

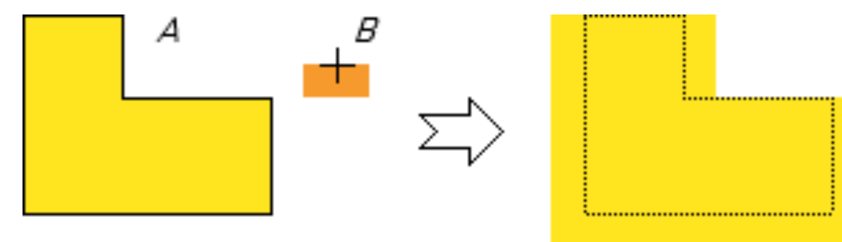


Dilation and Erosion in $\mathbb{R}^2$

- Structuring element

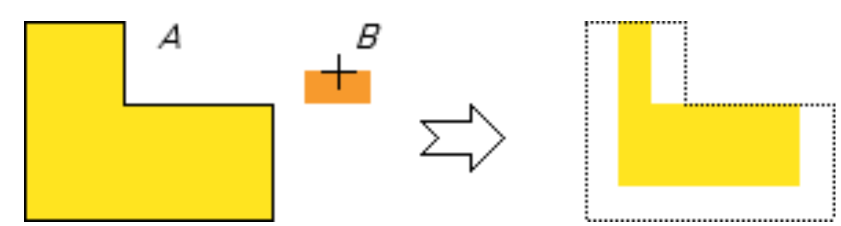
- Dilation

$$A \oplus B = \{x \in \mathbb{E} \;:\; (B^{\mathrm{s}})_x \cap A \neq \varnothing\} = \bigcup_{x \in A} (B)_x$$

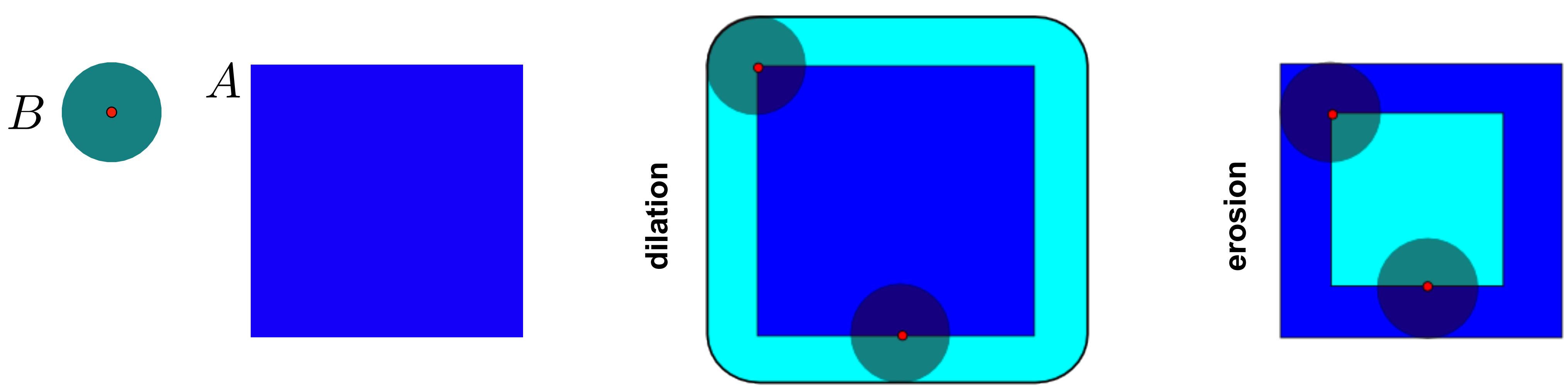


- Erosion

$$A \ominus B = \{x \in \mathbb{E} \;:\; (B)_x \subset A\} = \bigcap_{x \in B^{\mathrm{s}}} (A)_x$$



- Example



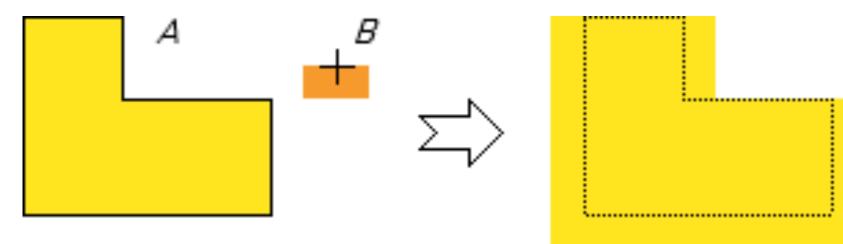
Dilation and Erosion in $\mathbb{R}^2$

- Structuring element B 

 B^{s}

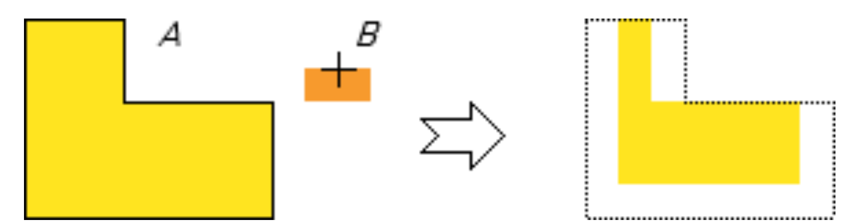
- Dilation

$$A \oplus B = \{x \in \mathbb{E} \,:\, (B^{\mathrm{s}})_x \cap A \neq \varnothing\} = \bigcup_{x \in A} (B)_x$$



- Erosion

$$A \ominus B = \{x \in \mathbb{E} \,:\, (B)_x \subset A\} = \bigcap_{x \in B^{\mathrm{s}}} (A)_x$$



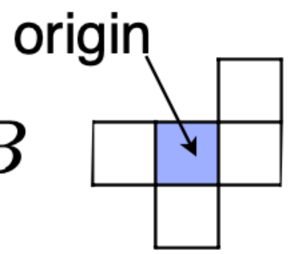
- Duality relations

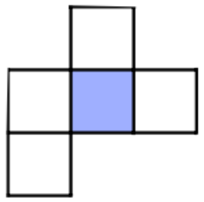
$$A \ominus B = (A^{\mathrm{c}} \oplus B^{\mathrm{s}})^{\mathrm{c}}$$ "Erosion = Dilation of complement"

$$A \oplus B = (A^{\mathrm{c}} \ominus B^{\mathrm{s}})^{\mathrm{c}}$$ "Dilation = Erosion of complement"

Dilation and Erosion in $\mathbb{Z}^2$

- Structuring element

B 

B^{s} 

- Dilation

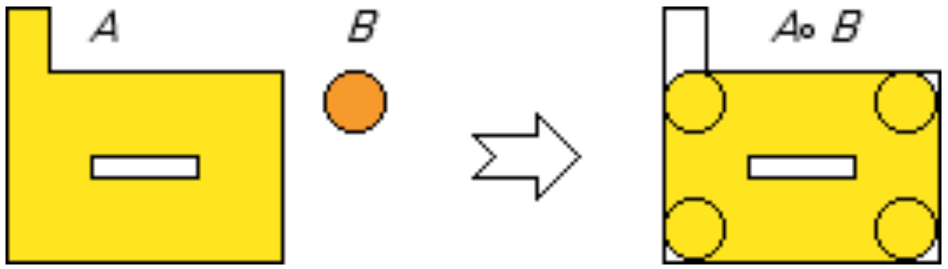
$$A \oplus B = \{x \in \mathbb{E} \,:\, (B^{\mathrm{s}})_x \cap A \neq \varnothing\} = \bigcup_{x \in A} (B)_x$$

- Erosion

$$A \ominus B = \{x \in \mathbb{E} \,:\, (B)_x \subset A\} = \bigcap_{x \in B^{\mathrm{s}}} (A)_x$$

Opening

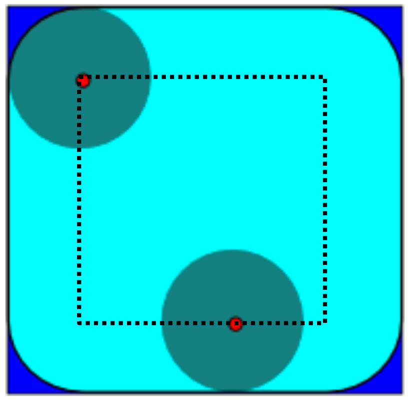
- Opening operator



$$A \circ B = (A \ominus B) \oplus B$$

  - Interpretation 1: "Smallest" set that contains a given erosion $A \ominus B$
  - Interpretation 2: Union of all shifted B's included in A

$$A \circ B = \bigcup_{x \in \mathbb{E}} \{(B)_x \ : \ (B)_x \subset A\}$$

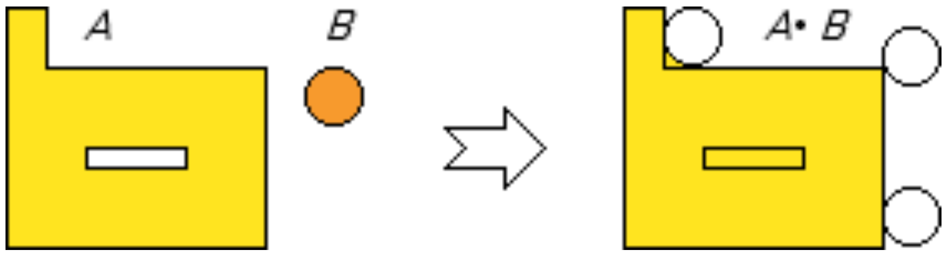


- Properties
  - Subset $\quad A \circ B \subset A$
  - Invariance to origin $\quad A \circ (B)_x = A \circ B$ for all $x \in \mathbb{E}$
  - Idempotence $\quad (A \circ B) \circ B = A \circ B$
  - Order preservation $\quad C \subset D \quad \Rightarrow \quad (C \circ B) \subset (D \circ B)$

Closing

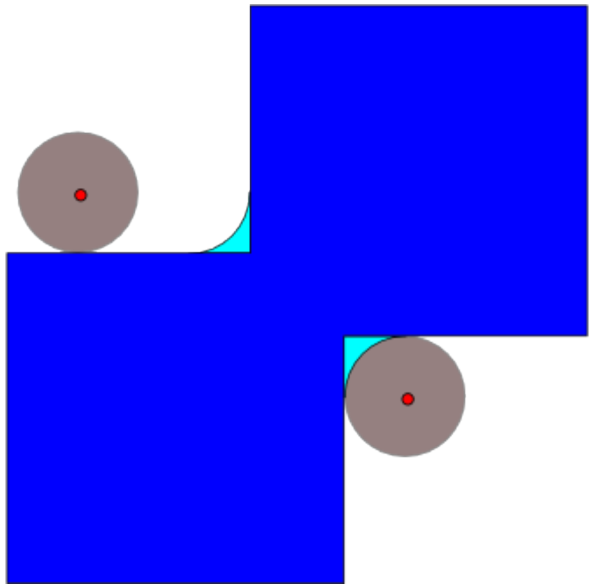
- Closing operator

  $$A \bullet B = (A \oplus B) \ominus B$$



  - Interpretation 1: "Largest" set that contains a given dilation $A \oplus B$
  - Interpretation 2: Complement of all shifted B^{s}'s included in A^{c}

  $$A \bullet B = \left(\bigcup_{x \in \mathbb{E}} \{ (B^{\mathrm{s}})_x \; : \; (B^{\mathrm{s}})_x \subset A^{\mathrm{c}} \} \right)^{\mathrm{c}}$$



- Properties
  - Superset $\quad A \bullet B \supset A$
  - Invariance to origin $\quad A \bullet (B)_x = A \bullet B$ for all $x \in \mathbb{E}$
  - Idempotence $\quad (A \bullet B) \bullet B = A \bullet B$
  - Order preservation $\quad C \subset D \quad \Rightarrow \quad (C \bullet B) \subset (D \bullet B)$

Duality Relations

- Erosion and dilation

$$A \ominus B = (A^{\mathrm{c}} \oplus B^{\mathrm{s}})^{\mathrm{c}}$$

$$A \oplus B = (A^{\mathrm{c}} \ominus B^{\mathrm{s}})^{\mathrm{c}}$$

- Opening and closing

$$A \circ B = (A^{\mathrm{c}} \bullet B^{\mathrm{s}})^{\mathrm{c}}$$

$$A \bullet B = (A^{\mathrm{c}} \circ B^{\mathrm{s}})^{\mathrm{c}}$$

Distance Map and Watershed

$$A_0[\mathbf{k}] = \begin{cases} 1, & \mathbf{k} \in \textbf{object} \\ 0, & \mathbf{k} \in \textbf{background} \end{cases}$$

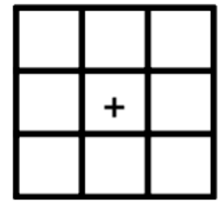
$n = 0$

while $\max(A_n) = 1$ **do**

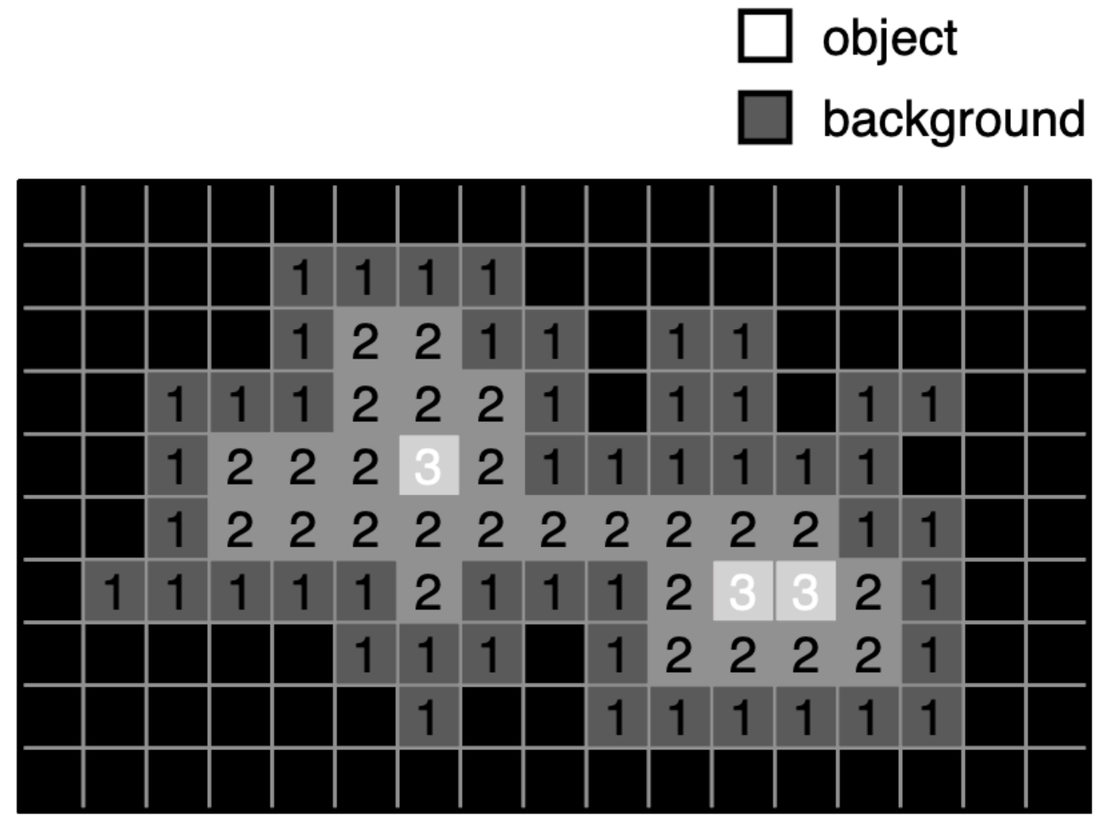
$A_{n+1} = \textbf{erode}(A_n, B)$

$D_{\text{map}} = D_{\text{map}} + (n+1)(A_n - A_{n+1})$

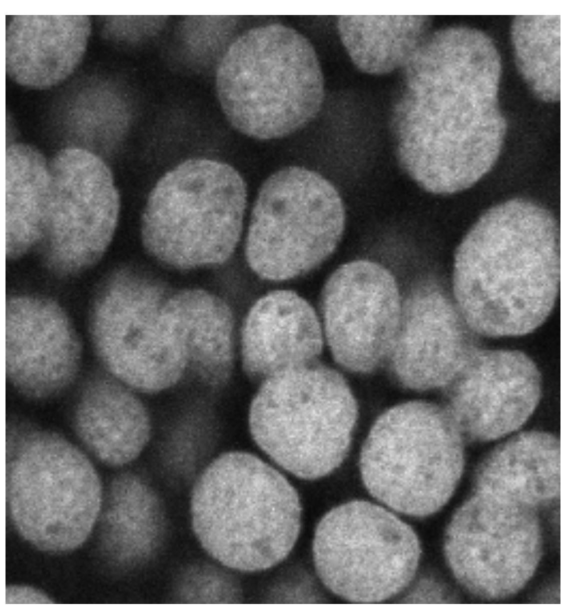
$n = n + 1$



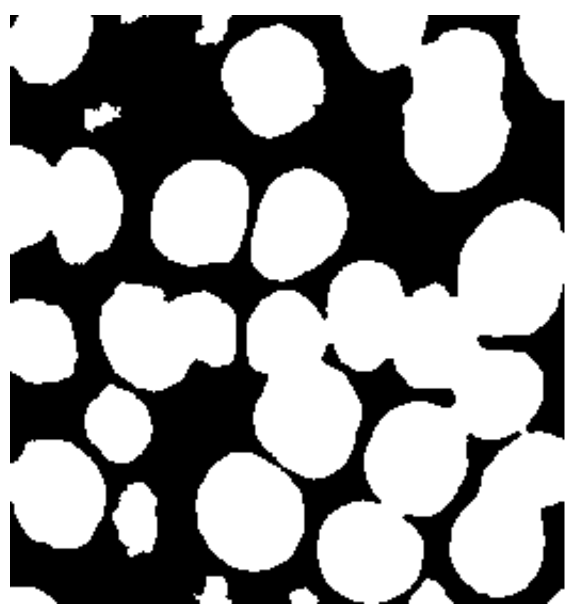
B

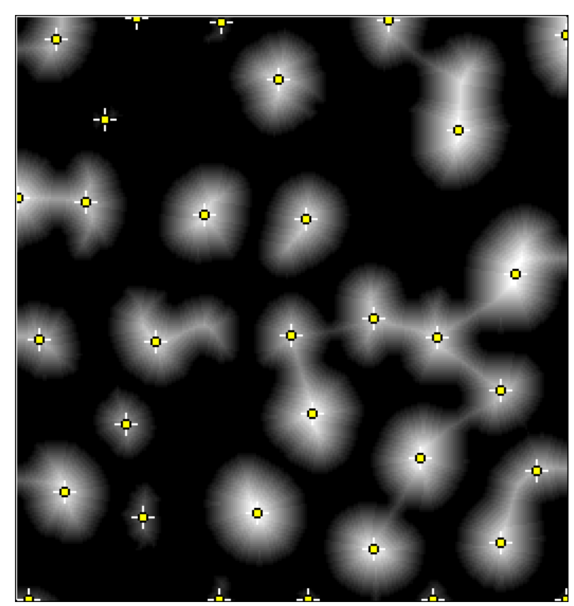


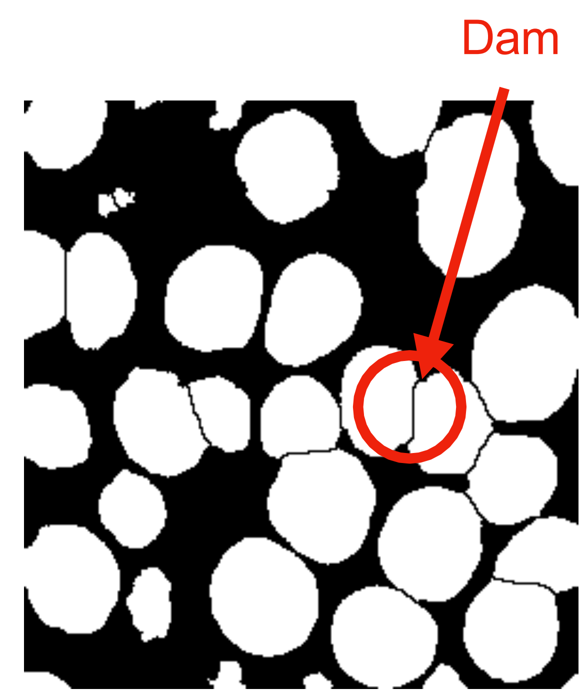
original image



thresholded image



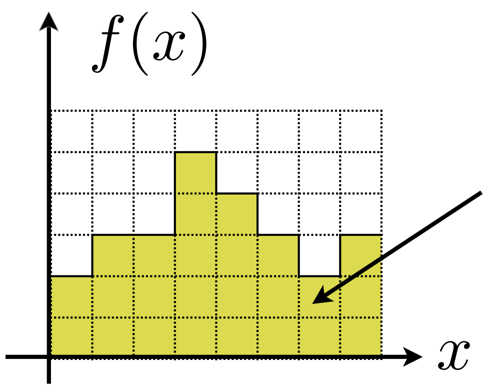




local maxima $\Rightarrow$ seeds of watersheds

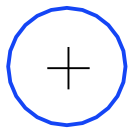
https://bigwww.epfl.ch/demo/ip/demos/watershed/

Graylevel Morphology

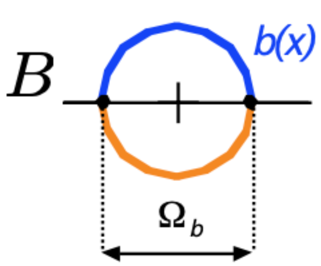


1D signal

Structuring elements are now **sequences**, e.g., $b[\boldsymbol{k}] = b[k_1, k_2]$

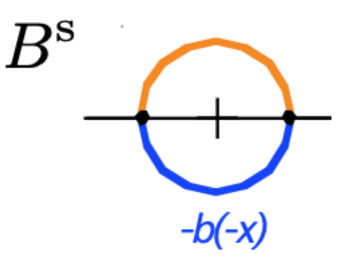
- Common structural elements are **symmetric**
  - Horizontal: W-neighborhood 
  - Volumetric: approximation of a ball (rolling ball) 
- Dilation

$$(f \oplus b)[\boldsymbol{k}] = \max_{\boldsymbol{k}_0 \in \Omega_b} \{f[\boldsymbol{k} - \boldsymbol{k}_0] + b[\boldsymbol{k}_0] \ : \ (\boldsymbol{k} - \boldsymbol{k}_0) \in \Omega_f\}$$



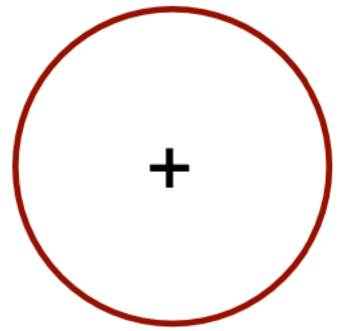
- Erosion

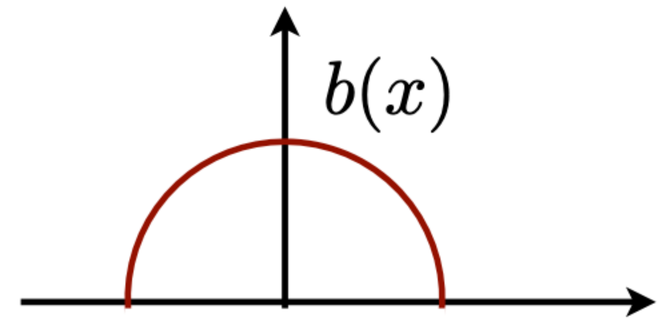
$$(f \ominus b)[\boldsymbol{k}] = \min_{-\boldsymbol{k}_0 \in \Omega_b} \{f[\boldsymbol{k} - \boldsymbol{k}_0] - b[-\boldsymbol{k}_0] \ : \ (\boldsymbol{k} - \boldsymbol{k}_0) \in \Omega_f\}$$



Graylevel Dilation and Erosion

Structuring element:

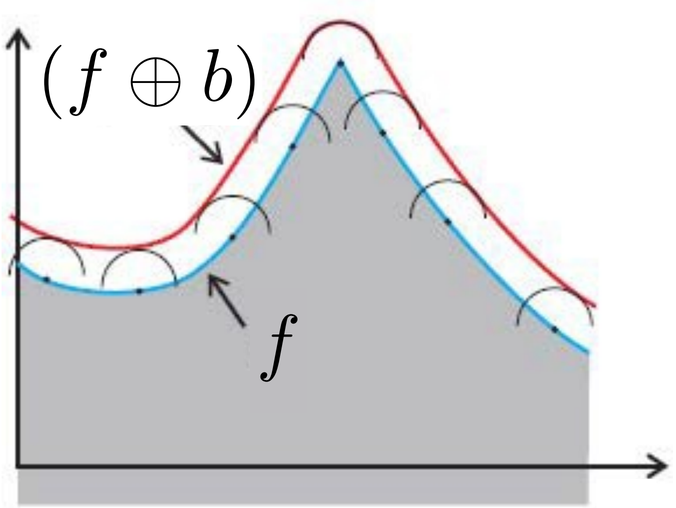


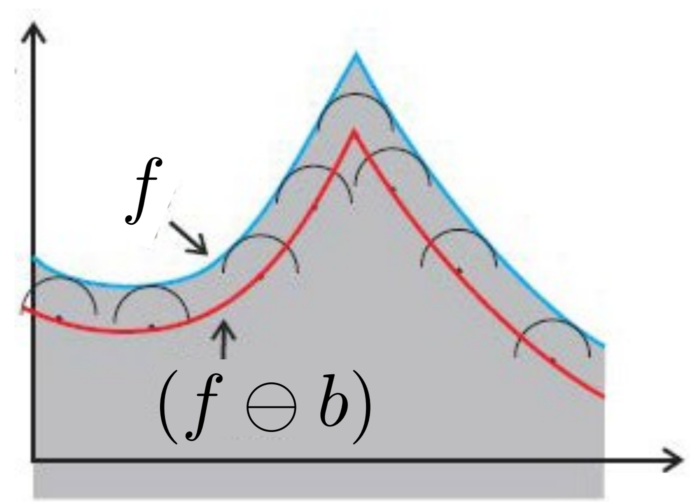


Reminder:

$$A \oplus B = \bigcup_{x \in A} (B)_x$$

$$A \ominus B = \{x \in \mathbb{E} \;:\; (B)_x \subset A\}$$





$$\max_{\boldsymbol{k}_0 \in \Omega_b} \{f[\boldsymbol{k} - \boldsymbol{k}_0] + b[\boldsymbol{k}_0] \;:\; (\boldsymbol{k} - \boldsymbol{k}_0) \in \Omega_f\}$$

$$\min_{-\boldsymbol{k}_0 \in \Omega_b} \{f[\boldsymbol{k} - \boldsymbol{k}_0] - b[-\boldsymbol{k}_0] \;:\; (\boldsymbol{k} - \boldsymbol{k}_0) \in \Omega_f\}$$

Graylevel Dilation and Erosion

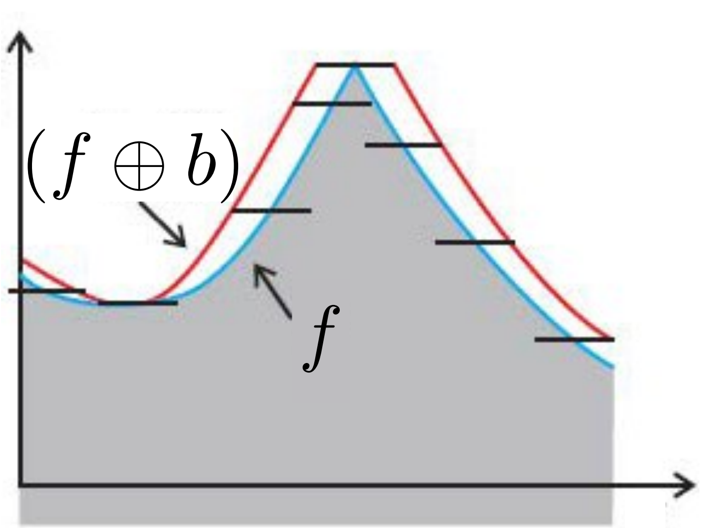
Structuring element:

$$b(x) = 0$$

$$\Omega_b = W$$

Reminder:

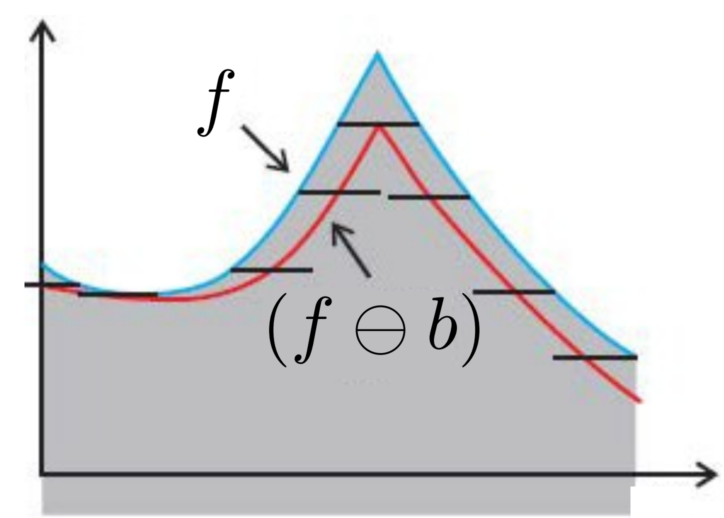
$$A \oplus B = \bigcup_{x \in A} (B)_x$$



$$\max_{\boldsymbol{k}_0 \in \Omega_b} \{ f[\boldsymbol{k} - \boldsymbol{k}_0] + b[\boldsymbol{k}_0] \ : \ (\boldsymbol{k} - \boldsymbol{k}_0) \in \Omega_f \}$$
$$= \max_{\boldsymbol{k}_0 \in W} \{ f[\boldsymbol{k} - \boldsymbol{k}_0] \ : \ (\boldsymbol{k} - \boldsymbol{k}_0) \in \Omega_f \}$$

max-filter

$$A \ominus B = \{ x \in \mathbb{E} \ : \ (B)_x \subset A \}$$



$$\min_{-\boldsymbol{k}_0 \in \Omega_b} \{ f[\boldsymbol{k} - \boldsymbol{k}_0] - b[-\boldsymbol{k}_0] \ : \ (\boldsymbol{k} - \boldsymbol{k}_0) \in \Omega_f \}$$
$$= \min_{-\boldsymbol{k}_0 \in W} \{ f[\boldsymbol{k} - \boldsymbol{k}_0] \ : \ (\boldsymbol{k} - \boldsymbol{k}_0) \in \Omega_f \}$$

min-filter

Morphological Filtering in Practice

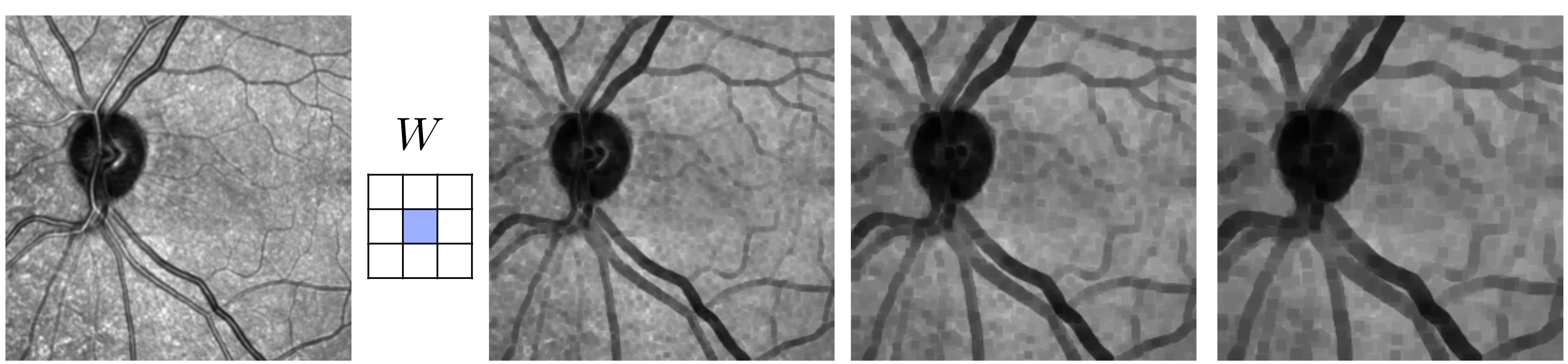
- Special case: $b[\boldsymbol{y}] = 0$ and $\Omega_b = W$

Dilation = Max-filter: $$\max_{\boldsymbol{k}_0 \in W} \{f[\boldsymbol{k} - \boldsymbol{k}_0] \ : \ (\boldsymbol{k} - \boldsymbol{k}_0) \in \Omega_f\}$$

Erosion = Min-filter: $$\min_{\boldsymbol{k}_0 \in W} \{f[\boldsymbol{k} - \boldsymbol{k}_0] \ : \ (\boldsymbol{k} - \boldsymbol{k}_0) \in \Omega_f\}$$

- Benefit of iteration

Dilation/erosion can be iterated to construct larger equivalent structuring elements

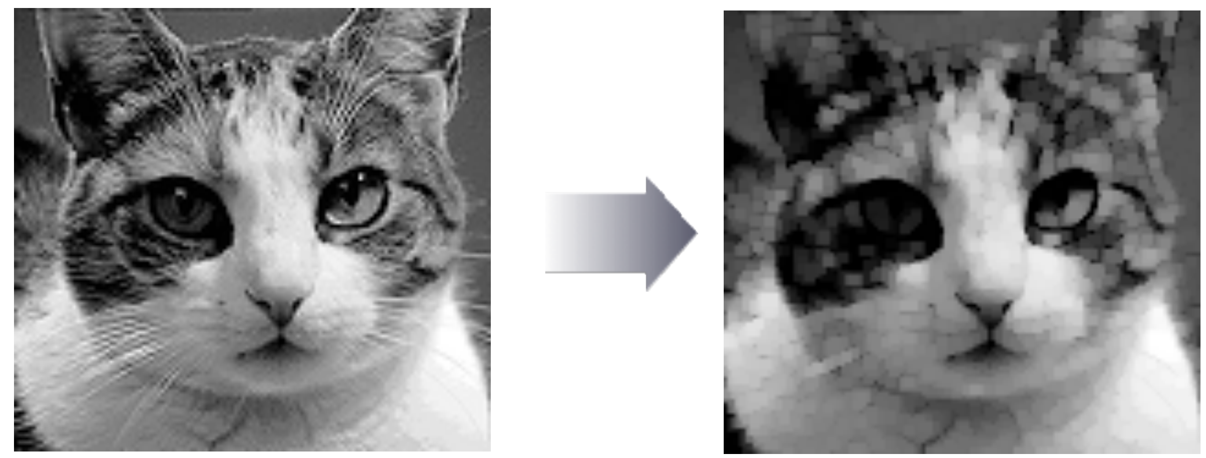


input 240×240 | 3×3 erosion | 2 iterations 5×5 erosion | 3 iterations 7×7 erosion

Morphological Filtering in Practice

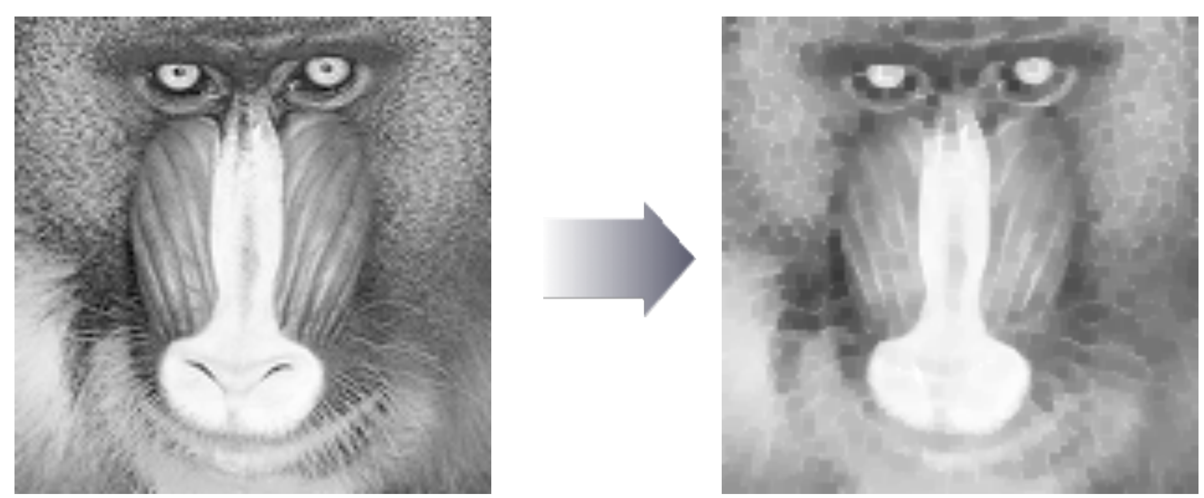
- Morphological smoothing
  - Opening (i.e., $\min$ then $\max$)

    $f \circ b = (f \ominus b) \oplus b$ Smoothing by suppression of small bright features

  - Closing (i.e., $\max$ then $\min$)

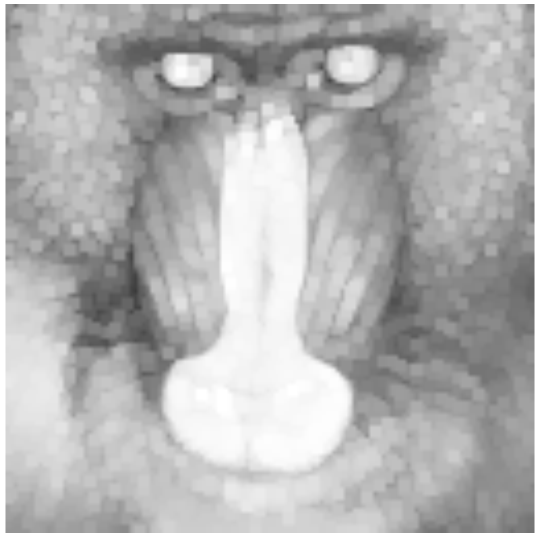
    $f \bullet b = (f \oplus b) \ominus b$ Smoothing by suppression of small dark features

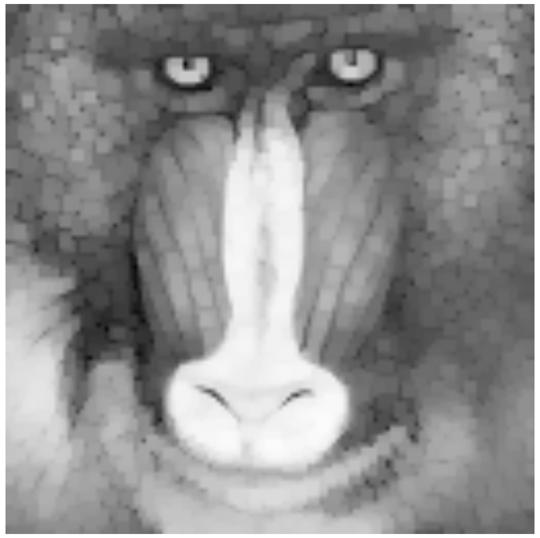
    

Morphological Filtering in Practice


Original (reduced): 128×128


A: 3×3 min


B: 3×3 max


C: 3×3 max of A


D: 3×3 min of B

Morphological Filtering

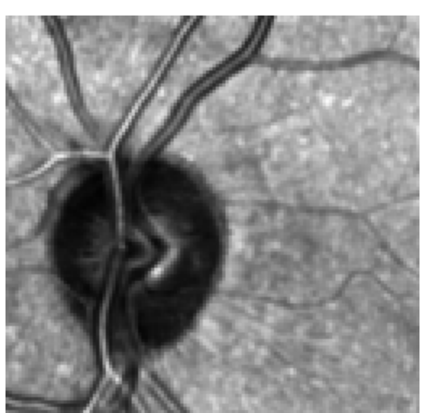
- Morphological gradient

$$g = (f \oplus b) - (f \ominus b)$$

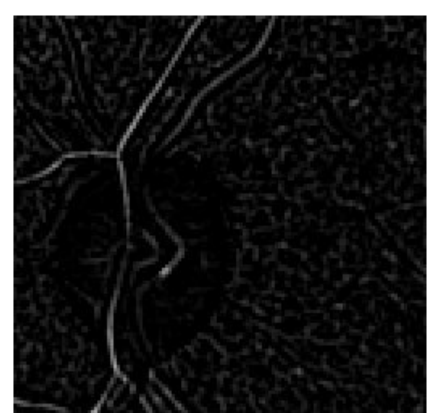
Property: not sensitive to edge direction when using symmetric b

- Top hat

Acts like the Laplacian

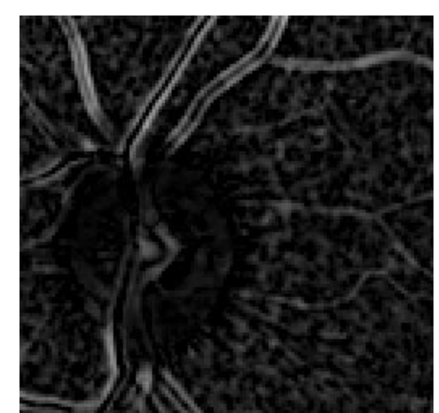


$g = f - (f \circ b)$



disk, $r = 1$

$g = (f \bullet b) - f$



disk, $r = 3$