ECE 172A: Introduction to Image Processing
Image Processing Tasks: Part II

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Outline

- Preprocessing ✓
  - Histogram
  - Normalization
  - Combining Images
  - Spatial Averaging
- Matching and Detection ✓
  - Correlation
  - Matched Filtering
- Feature Extraction ✓
  - Contour/Edge Detection
- Segmentation
  - Variational Thresholding
  - Connected-Component Labeling

Image Segmentation

- Segmentation: Art or Science?
- Amplitude Thresholding
  - Variational Thresholding
  - Statistical Thresholding
- Binary Segmentation Techniques

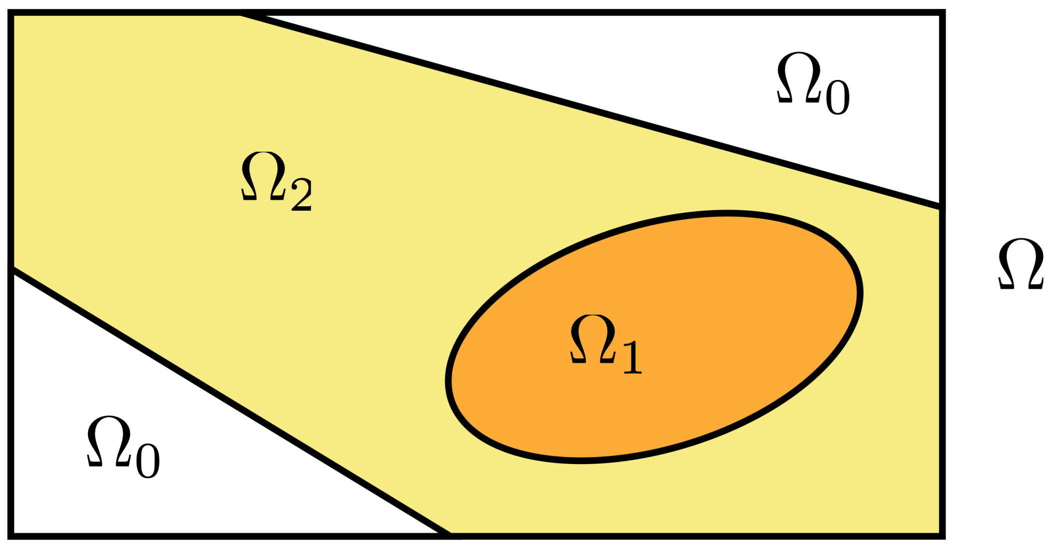
What is Segmentation?

- Definition

Image $f[\boldsymbol{k}]$, with $\boldsymbol{k} \in \Omega$

Image segmentation: Find a partition of the support Ω of the image f, with

$$\Omega = \bigcup_i \Omega_i \quad \text{with} \quad \Omega_i \cap \Omega_j = \varnothing \text{ for } i \neq j$$



such that the regions Ω_i satisfy some homogeneity (and connectivity) criterion.

The total number of regions is not necessarily known

- Three main approaches (not based on deep learning)
  - Pixel classification
  - Region-based segmentation
  - Boundary-based segmentation $\Rightarrow$ Edge detection

Segmentation: Art or Science?

Problem: lack of a universal definition of homogeneity
$\Rightarrow$ many application-specific approaches

- Approaches for specifying homogeneity
  - Empirical (e.g., similar graylevels; feature maps)
  - Statistical, based on some a priori model
    (e.g., constant mean + additive white noise)
- Approaches for enforcing connectivity (if required)
  - Prior information about object size or shape
  - Joint probability model for class labels
  - Contour length

Segmentation as an Optimization Problem

Variational vs. Statistical Approaches

Principle: Maximize the quality of any candidate segmentation, as measured by a functional that incorporates all problem-specific knowledge

$$\text{Criterion} = \text{data-fidelity} + \text{regularization}$$

homogeneity ↔ data-fidelity

connectivity ↔ regularization

Amplitude Thresholding

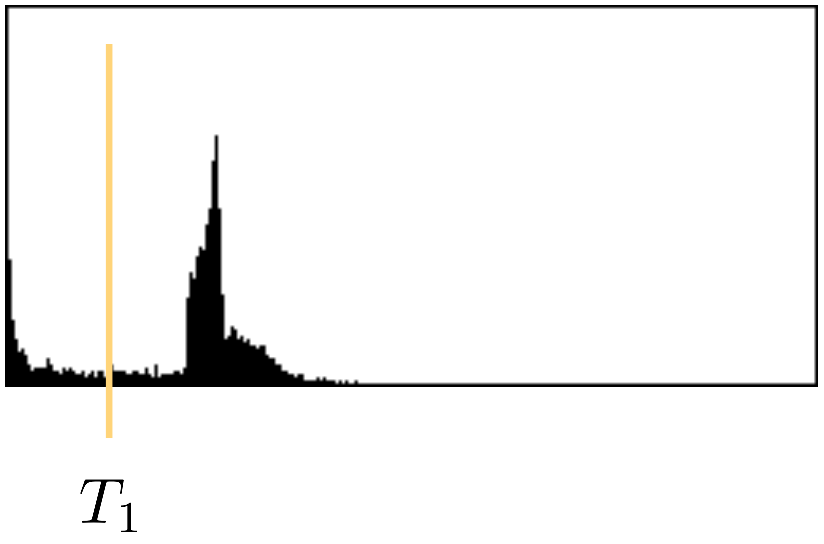
- Empirical approach

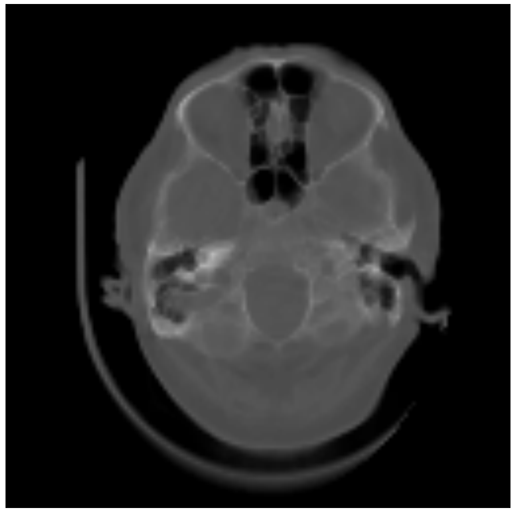
Based on the histogram, select a collection of thresholds

$$T_1 < \cdots < T_i < \cdots < T_I$$

and use the following rule to assign regions:

$$\boldsymbol{k} \in \Omega_i \text{ for } T_i \leq f[\boldsymbol{k}] \leq T_{i+1}$$





Variational Thresholding

Principle: Minimize an appropriate goodness-of-fit criterion

- Variational formulation

  Constant-mean model: $f[\boldsymbol{k}] = \mu_i$, $\boldsymbol{k} \in \Omega_i$

  Find μ_i and Ω_i such that $\displaystyle\sum_i \sum_{\boldsymbol{k}\in\Omega_i} (f[\boldsymbol{k}] - \mu_i)^2$ is minimum

  $\Rightarrow$ same problem as Llody–Max quantization (K-means)

Simple iterative two-step optimization scheme

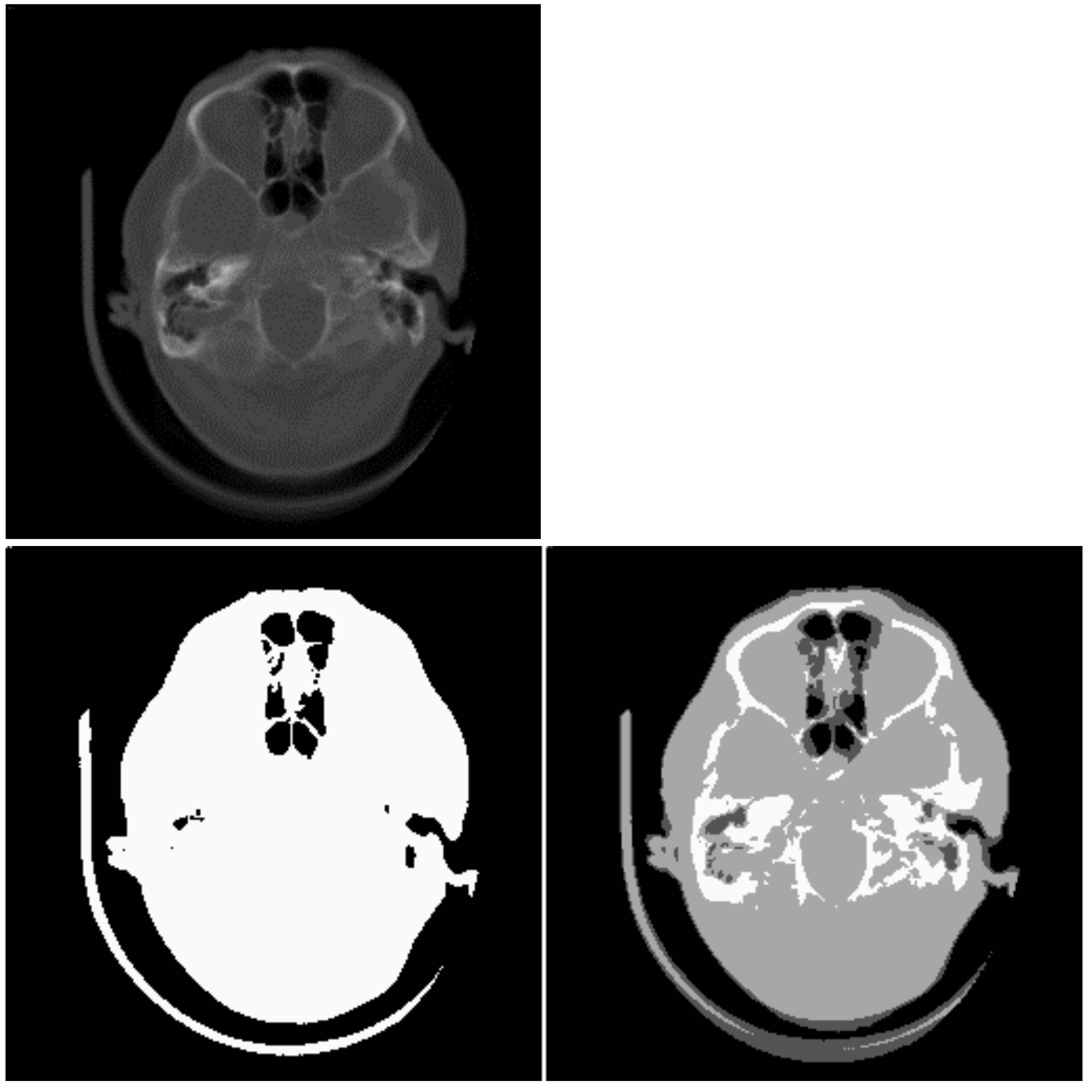
1. Given Ω_i, compute region means μ_i
2. Given μ_i, compute optimal partitions $\Omega_1, \ldots, \Omega_I$
   $\Rightarrow \quad T_{i+1} = \frac{1}{2}(\mu_i + \mu_{i+1})$

Note: all computations can be done from the histogram

Segmentation by Minimum-Error Quantization

Search for the “optimal” threshold values to segment images

Minimum mean squared error solutions:



$K = 2$ $K = 4$

Statistical Thresholding

Principle: Find the most “likely” segmentation model

- Statistical formulation

  For every pixel $\boldsymbol{k}$, we have the label $\ell[\boldsymbol{k}] = i \Leftrightarrow \boldsymbol{k} \in \Omega_i$

  $f[\boldsymbol{k}]$, $\boldsymbol{k} \in \Omega$: pixel graylevels

  Typical assumption:
  Conditional density of a pixel graylevel given a segment label is **Gaussian**

$$p(f[\boldsymbol{k}] \mid \ell[\boldsymbol{k}] = i) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(f[\boldsymbol{k}] - \mu_i)^2}{2\sigma^2}\right)$$

Parameters: $\sigma > 0$, $\mu_1, \ldots, \mu_K \in \mathbb{R}$

for all $\boldsymbol{k} \in \Omega$

Under this statistical model, we want to find the most likely segmentation

Statistical Thresholding (cont'd)

Assume: each pixel graylevel is independent of all the others

$f[\boldsymbol{k}_1]$ is independent of $f[\boldsymbol{k}_2]$ when $\boldsymbol{k}_1 \neq \boldsymbol{k}_2$

Collect: All pixel graylevels into one random vector $\boldsymbol{F} = \{f[\boldsymbol{k}]\}_{\boldsymbol{k}\in\Omega}$

Exercise: What is the density of $\boldsymbol{F}$ given $\{\ell[\boldsymbol{k}]\}_{\boldsymbol{k}\in\Omega}$?

$$p(\boldsymbol{F} \mid \{\ell[\boldsymbol{k}]\}_{\boldsymbol{k}\in\Omega}) \propto \prod_{\boldsymbol{k}\in\Omega} \exp\left(-\frac{(f[\boldsymbol{k}] - \mu_{\ell[\boldsymbol{k}]})^2}{2\sigma^2}\right)$$

Goal: Find parameters $\ell[\boldsymbol{k}]$ and $\mu_1, \ldots, \mu_K$
to maximize the "likelihood" of $\boldsymbol{F}$

Find μ_i and $\ell[\boldsymbol{k}]$ such that $\displaystyle\sum_i \sum_{\boldsymbol{k}\in\Omega_i} (f[\boldsymbol{k}] - \mu_i)^2$ is minimum

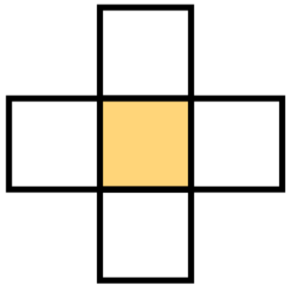
Equivalent to Lloyd–Max quantization!

Binary-Segmentation Techniques

Objects or regions: Sets of points in $\mathbb{Z}^2$ (bitmap)

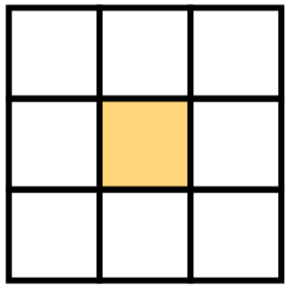
- Distances between $\boldsymbol{a}, \boldsymbol{b} \in \mathbb{Z}^2$
  - City-block distance: $D_4(\boldsymbol{a}, \boldsymbol{b}) = |a_1 - b_1| + |a_2 - b_2|$ — ℓ^1-distance: $\|\boldsymbol{a} - \boldsymbol{b}\|_1$
  - Chessboard distance: $D_8(\boldsymbol{a}, \boldsymbol{b}) = \max\{|a_1 - b_1|, |a_2 - b_2|\}$ — ℓ^∞-distance: $\|\boldsymbol{a} - \boldsymbol{b}\|_\infty$
- Connectivity
  - 4-connected neighborhood

$$N_4(\boldsymbol{a}) = \{\boldsymbol{b} \in \mathbb{Z}^2 \;:\; D_4(\boldsymbol{a}, \boldsymbol{b}) \leq 1\}$$



  - 8-connected neighborhood

$$N_8(\boldsymbol{a}) = \{\boldsymbol{b} \in \mathbb{Z}^2 \;:\; D_8(\boldsymbol{a}, \boldsymbol{b}) \leq 1\}$$

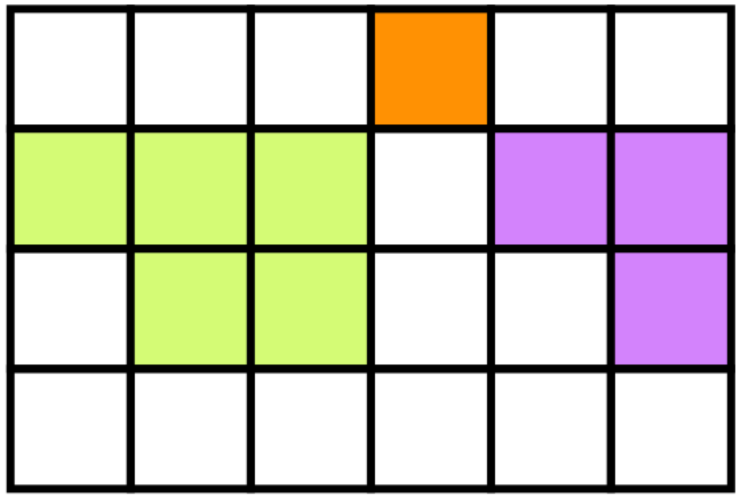


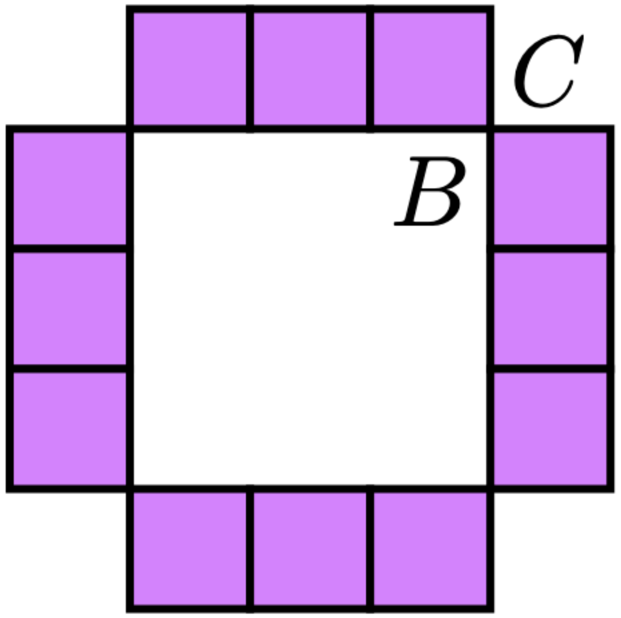
$\mathbb{Z}^3 \Rightarrow$ 6-connected, 18-connected, or 26-connected neighborhoods

Binary-Segmentation Techniques (cont'd)

Objects or regions: Sets of points in $\mathbb{Z}^2$ (bitmap)

- Path
  - List $\{\boldsymbol{a}_i : i = 1, \ldots, M\}$ of M connected pixels such that $\boldsymbol{a}_i \in N(\boldsymbol{a}_{i-1})$
- Connected components
  - Maximal set of connected pixels





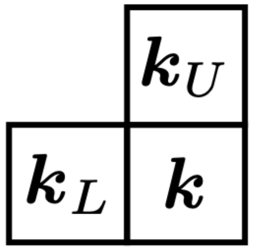
Connected-Component Labeling

- Connected-component labeling (blob coloring) algorithm

$\mathbb{F}$: foreground
$\mathbb{B}$: background
$\mathbb{E}$: color equivalences

$\mathbb{Z}^2 = \mathbb{F} \cup \mathbb{B}$; $\mathbb{F} \cap \mathbb{B} = \varnothing$

4-connected scanning window



Initialize $\mathbb{E} = \varnothing$ and initial color $i = 1$

Scan image from left to right then top to bottom

if $\boldsymbol{k} \in \mathbb{F}$ then {

  if $(\boldsymbol{k}_U \in \mathbb{B} \wedge \boldsymbol{k}_L \in \mathbb{B})$ then $\{color(\boldsymbol{k}) = i;\ \mathbb{E} = \mathbb{E} \cup \{(i, i)\};\ i = i+1;\}$

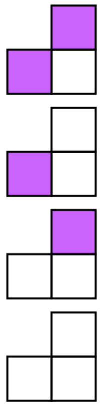
  if $(\boldsymbol{k}_U \in \mathbb{F} \wedge \boldsymbol{k}_L \in \mathbb{B})$ then $color(\boldsymbol{k}) = color(\boldsymbol{k}_U);$

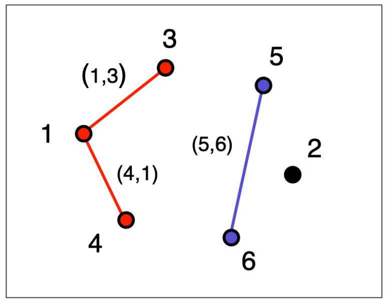
  if $(\boldsymbol{k}_U \in \mathbb{B} \wedge \boldsymbol{k}_L \in \mathbb{F})$ then $color(\boldsymbol{k}) = color(\boldsymbol{k}_L);$

  if $(\boldsymbol{k}_U \in \mathbb{F} \wedge \boldsymbol{k}_L \in \mathbb{F})$ then {

    $color(\boldsymbol{k}) = color(\boldsymbol{k}_L);\ \mathbb{E} = \mathbb{E} \cup \{(color(\boldsymbol{k}_U), color(\boldsymbol{k}_L))\};\}$

}

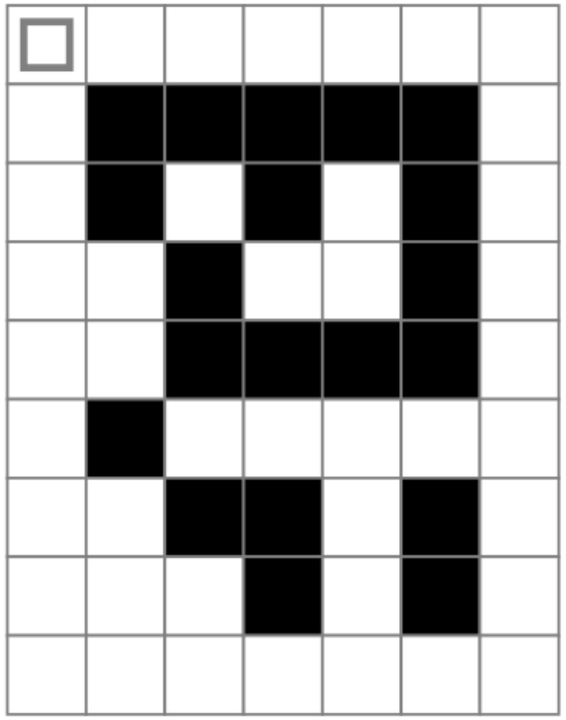




- Post-processing: Resolve color equivalences

e.g., $\mathbb{E} = \{(1,1), (2,2), (3,3), (1,3), (4,4), (4,1), (5,5), (6,6), (5,6)\}$

Connected-Component Labeling Exercise



white is background
black is foreground

$$\texttt{if}\ \boldsymbol{k} \in \mathbb{F}\ \texttt{then}\ \{$$

$$\texttt{if}\ (\boldsymbol{k}_U \in \mathbb{B} \wedge \boldsymbol{k}_L \in \mathbb{B})\ \texttt{then}\ \{\mathit{color}(\boldsymbol{k}) = i;\ \mathbb{E} = \mathbb{E} \cup \{(i,i)\};\ i = i+1;\}$$

$$\texttt{if}\ (\boldsymbol{k}_U \in \mathbb{F} \wedge \boldsymbol{k}_L \in \mathbb{B})\ \texttt{then}\ \mathit{color}(\boldsymbol{k}) = \mathit{color}(\boldsymbol{k}_U);$$

$$\texttt{if}\ (\boldsymbol{k}_U \in \mathbb{B} \wedge \boldsymbol{k}_L \in \mathbb{F})\ \texttt{then}\ \mathit{color}(\boldsymbol{k}) = \mathit{color}(\boldsymbol{k}_L);$$

$$\texttt{if}\ (\boldsymbol{k}_U \in \mathbb{F} \wedge \boldsymbol{k}_L \in \mathbb{F})\ \texttt{then}\ \{$$

$$\mathit{color}(\boldsymbol{k}) = \mathit{color}(\boldsymbol{k}_L);\ \mathbb{E} = \mathbb{E} \cup \{(\mathit{color}(\boldsymbol{k}_U), \mathit{color}(\boldsymbol{k}_L))\};\}$$

$$\}$$

How many 4-connected components are there?